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if we adopt $S = 1{\cdot}75 \times 10^6$. This shows that if ρ is about 6, or something like the mean density of the Earth, the diameter of the particles composing the comet is of the order of 3 mm. This result confirms the hypothesis as to the physical constitution of comets on which this investigation rests, and supplies the first numerical estimate of the size of the meteoric bodies which constitute a cometary swarm.

8. Dr Svedstrup's discovery amply justifies the statement before made, that "it is unwarrantable to *assume* that the mean motion and the mean distance are related and not independent elements." This must be borne in mind in all future discussions of the definitive orbits of comets; otherwise a valuable means of gaining an insight into the constitution of these bodies will be neglected. It now appears exceedingly probable that in this direction lies the clue to the explanation of the anomalous motion of Encke's Comet, as suggested in § 3 of my earlier paper.

University Observatory, Oxford:
1906 *August* 18.

Note on a Mechanical Solution of Kepler's Equation.

By H. C. Plummer, M.A.

This note refers to the method of obtaining an approximate solution of Kepler's equation which was proposed by Dr Rambaut (*Monthly Notices*, vol. l. p. 301), and was subsequently described independently by myself (*Monthly Notices*, vol. lvi. p. 317), and again later by Herr B. Gonggrijp (*Ast. Nach.*, 3720). The principle of the method, which is based on the use of a trochoid, is to be found in Newton's *Principia*, book i. prop. 31.

Quite recently Dr Rambaut has proposed a new method which combines great mathematical elegance with considerable practical advantages (*Monthly Notices*, vol. lxvi. p. 519). His paper suggests that it may not be out of place if I describe some modifications in the application of the trochoid principle which occurred to me, by a curious coincidence, about the same time.

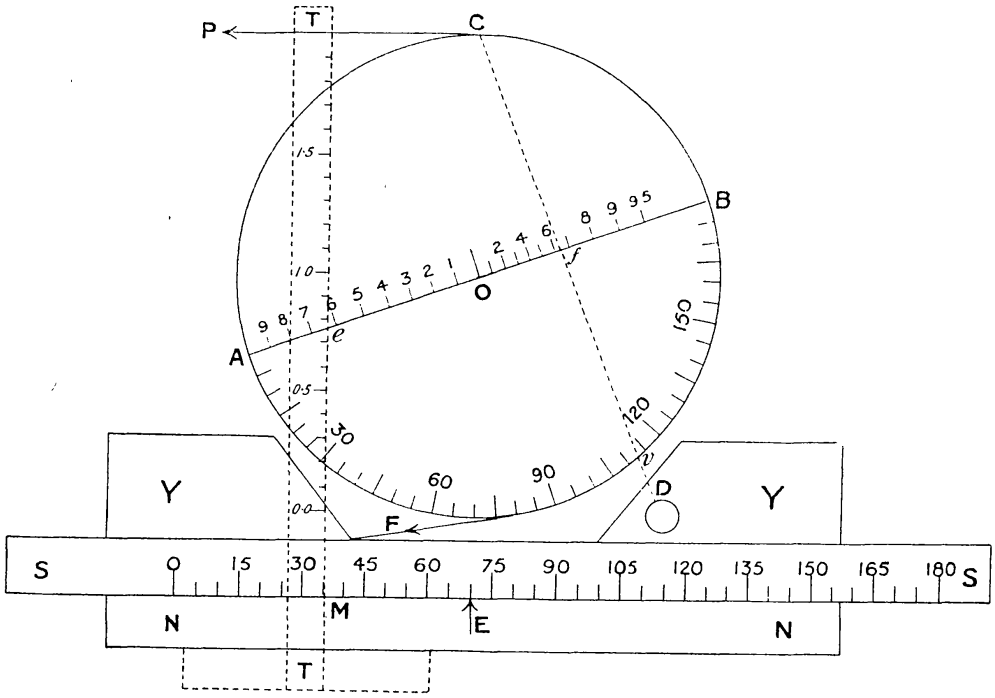
In the accompanying figure, A B C is a flat circular disc of suitable radius, mounted on a board so as to be capable of turning about its centre O. Fixed to the board and flush with the surface of the disc are wooden pieces Y Y, N N, which serve as guides to a scale S S, which, with bevelled edges, can slide between them. The point E, marked with a pointer, is the foot of the perpendicular from O on the lower edge of the scale. Attached near B, and passing round the edge of the disc, is a thin flexible metallic tape F, the other end of which is attached to the end of the scale S S. This tape is kept taut by another tape B C P passing round the disc in the opposite direction and fastened to a weak spring or, much better, to a hanging weight.

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The rotation of the disc thus depends on and can be measured by the motion of the scale S S, a part of which, in length equal to half the circumference of the disc, is divided into 180° in such a way that the origin coincides with the point E when the radius O A is perpendicular to the scale. Then clearly the pointer E at any time marks the angle (< 180°) through which the disc has been turned from its initial position.

The initial radius O A is divided in fractions of itself (as shown, in tenths), so that a length O*e* can be taken corresponding to the eccentricity, the whole radius being unity. The lower edge of the guide N N is parallel to the scale S S, so that the edge of a T-square



T T can be kept perpendicular to the scale. The instrument can now be used to find an approximate solution of Kepler's equation, according to the following rule:—

Place the edge of the T-square at the reading M *on the scale* S S. *Move the scale and T-square together till the point e on* O A *which marks the eccentricity e falls on the edge of the T-square. Then read the position of the pointer* E *on the scale. The reading is the eccentric anomaly* E *corresponding to the mean anomaly* M.

For clearly

$$E - M = e \sin E \quad . \quad . \quad . \quad . \quad (1)$$

If M *lies between* 180° *and* 360°, *use the instrument according to the above rule to find* 360° − E *from* 360° − M.

The figure is drawn for $e = 0{\cdot}65$, showing M = 35°·0 and E = 70°·0. The rolling disc of the device formerly described, with its troublesome manipulation and liability to error, is now eliminated. The present instrument is nearly as simple to use as a slide-rule, and

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it has the great advantage of being direct-reading. Two straight, uniformly divided scales O A and S S alone are necessary; and the accuracy of reading can be increased by placing a vernier instead of a simple pointer at E, and attaching another vernier to the T-square at M. The surface of the scale S S should be very slightly above the level of Y Y, N N and the disc. The T-square can then be kept at a fixed reading with one hand while the scale is being moved by the other. It would be still better if the head of the T-square moved along a groove in the scale itself. In such details of construction there is naturally much room for variety, but it is possible to make an instrument on these general lines which is both simple and efficient.

There are, however, three additional scales shown in the figure which may be briefly explained. The first is on the edge of the T-square, which is graduated in fractions of the radius of the disc. The origin of this scale is on a level with the lowest point of the disc. It is thus possible to read directly at the point e the height of this point above the lowest point of the disc, and this reading clearly corresponds to the quantity

$$1 - e\cos \mathrm{E} = \frac{r}{a} = \frac{d\mathrm{M}}{d\mathrm{E}} \quad . \quad . \quad . \quad (2)$$

This is the ratio of the radius vector to the mean distance, but its more important significance arises from the fact that it is the ratio of corresponding increments of the mean and eccentric anomalies. Thus when an approximate E (as found by mechanical means or otherwise) is known, and is found to lead to an error dM in M, the approximate error in the assumed E is found by dividing dM by the reading on the edge of the T-square at the point where it is crossed by the radius OA of the disc. In the case represented in the figure this reading should be 0·778.

Up to this point we have considered only practical modifications in applying the principles of a device formerly described. The graduation of one semicircle and the unequally divided scale on the radius O B, as shown in the figure, are due to an addition which, whatever its value, appears to be new. The object of this is to obtain by a direct reading an approximate value of the true anomaly. The graduations in the radius O B (numbered in the figure for 10 e) are made in such a way that corresponding to any eccentricity e a point f can be read off such that

$$Of/\mathrm{OB} = \tan \tfrac{1}{2}\phi \quad \text{where} \quad \sin\phi = e$$

A thread C D is attached to the point C, the highest point of the disc. The disc being in the position already found, the thread is stretched over the point f and crosses the edge of the disc at v. Then the angle A O v or v is such that

$$\tan \tfrac{1}{2}v = \sqrt{\frac{1+e}{1-e}}\tan \tfrac{1}{2}\mathrm{E} \quad . \quad . \quad . \quad (3)$$

so that v is the true anomaly corresponding to the mean anomaly M.

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The geometrical principle on which this very simple device is based need not be proved here.* An accurate value of the true anomaly can only be found by calculation from an accurate value of E. But there may be cases in which only a rough value is required, and in any case a value of this kind, when so easily obtained, may be useful in preventing any serious slip in the calculation from being overlooked. In the case represented in the figure the value of v should be $113^{\circ}\cdot3$.

The complete instrument thus provides the means of finding, for any value of the eccentricity between 0·1 and 0·9, approximate values of the eccentric and true anomalies and of the radius vector, when the mean anomaly or time is given. It thus solves in an approximate manner the whole problem of elliptic motion as expressed by the relations (1), (2), and (3), and the ease with which this can be done suggests that the instrument may have some slight educational as well as practical value.

1906 *September* 18.

Solar Parallax Papers. No. 5.

Examination of the Photographic Places of Stars published in the Paris Eros Circulars. By Arthur R. Hinks, M.A.

1. In a preceding paper, *Solar Parallax Papers, No.* 4 (*Monthly Notices*, 1906 June, lxvi. p. 481), it was shown that the photographic right ascensions of *repère* stars, published in Paris circulars 10 and 11, had little if any magnitude equation special to each observatory (with one exception); and the conclusion was drawn that it is unlikely that they have an absolute magnitude equation common to all. This applies only to stars down to a magnitude little fainter than 9th. In the present paper I propose to extend the search for photographic magnitude equation to the declinations of the *repère* stars, and to the fainter stars in both co-ordinates; and to look for systematic errors other than those depending on magnitude, so far as they can be detected by intercomparison of published results.

2. The stars measured upon Eros plates in accordance with M. Loewy's programme are divided into three classes:—

i. The *étoiles de repère*, extending right up to the edges of plates 2° square, and generally in the outer regions rather than near the centre.

ii. The *étoiles de comparaison*, which have been used as comparison stars in visual micrometric observations. They lie mostly in a narrow belt along the track of the planet, and extend right up to two edges of the plate.

* The principle is the same as that employed in "A Method of Mechanically Compensating the Rotation of the Field of a Siderostat" (*M.N.*, vol. lxi. p. 402). See § 2 for a proof which can be adapted without difficulty to the present case, and §§ 8 and 9 for an examination of the geometry of the principle.

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